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CHALLENGES IN PARAMETER ESTIMATION FOR CONDITION ASSESSMENT OF STRUCTURES

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ABSTRACT

This paper presents an overview of current challenges in the field of parameter estimation with a specific application to condition assessment of structures. Over the past 10 years, a large number of methods have been proposed to monitor structural performance. Based on deviations of the analytical model from the observed mechanical behavior the changes in the structural parameters are then estimated. Using these parameter estimates, the condition of the structure is diagnosed. Many of these methods have shown promise using simulated or laboratory data, and some have even been successful when applied in a field test. However, the use of parameter estimation techniques for structural condition monitoring has difficulties to become practical for implementation in operating structures. Many of the reasons are economic, but many aspects of this technology are still in need of development in order to make the benefits of this technology apparent to the civil engineering community. This paper presents some of the issues, both technical and economical, that must be addressed in order to make parameter estimation for structural condition assessment a practical reality.

INTRODUCTION AND LITERATURE REVIEW

Parameter estimation is commonly referred to as system identification or structural identification. For this purpose, an a priori parametric analytical model that describes the response of a physical system is assumed, and then the system's actual response is measured experimentally. Through some correlation of the analytical model with the measured data the parameters of the analytical model are updated.

Parameter estimation using non-destructive test (NDT) data can be classified into problems with and without physical models. The physical models are either based on a modal model or a finite element model. NDT measurements can be displacements, rotations, strains, or dynamic response of structure in time or frequency domains. Parameter estimation with no physical models includes neural networks, ARMA modeling, and pattern recognition. This category includes those models that are purely mathematical constructs, with no particular relevance to structural mechanics. The focus of this paper is physical-based parameter estimation. The next three sections present a qualitative discussion of the most important challenges that limit the practical application of parameter estimation in the condition assessment of large civil-engineered structures.

Physical-based model identification can be either *modal parameter estimation* or *geometric parameter estimation*. One of the most common parameter identification methods in the field of structural dynamics is *modal parameter estimation*, as described by Ewins (1985). In this process the structure's response is assumed to be accurately described by standard discrete linear second order differential equations of motion. Using dynamic response of the system, modal parameters (e.g., natural frequency, mode shapes, and modal damping) are estimated. System parameters for such a model can be described in terms of mass, stiffness, and damping matrices as reviewed by Farrar et al. (1994) and Doebling et al. (1996). Geometric parameter estimation is the art of reconciling an FEM-based response with measured NDT data using optimization techniques. The analytical response used may be modal (as described above) or static measurements (Sanayei et al., 1986; 1998) to estimate the structural parameters at the component level. The advantage of geometric parameter estimation is that it can easily lend itself to damage assessment of structures.

Figure 1 shows typical loading conditions and outputs of the parameter estimation system. Structural responses to controlled or uncontrolled excitations are combined with an *a priori* analytical model via parameter estimation to produce either structural modal parameter estimates or structural geometric parameter estimates. Estimated modal properties also can be used to estimate the physical parameters. The use of operational loads such as truck loading, wind loading, or machine-induced vibrations provide a more practical approach to parameter estimation.

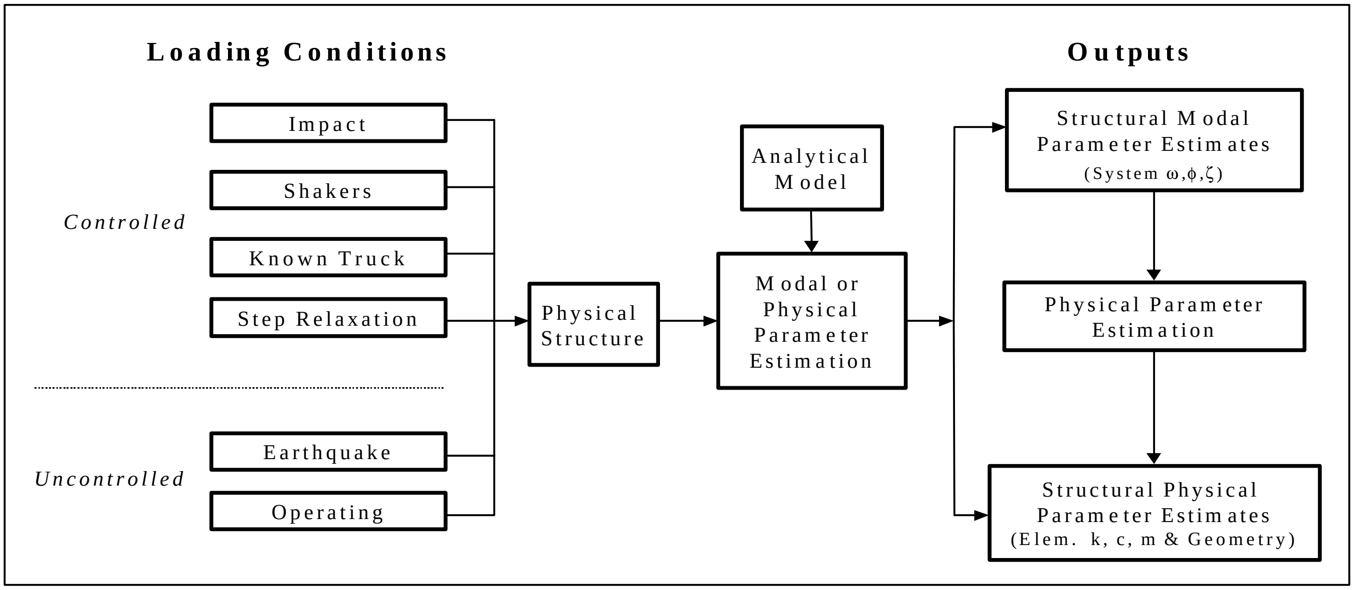

Figure 1: Overview of parameter estimation process

A more recent engineering application requiring parameter estimation, which is still in a research stage, is the use of changes in measured structural response as an indicator of structural condition or health (Doebling et al., 1996). Condition assessment is a relative concept requiring structural measurements to be made at two different times for comparison. Parameter estimation, on the other hand, is a temporally absolute concept; i.e. one measurement made at any time can be used to obtain an indication of the model accuracy at that time.

To the authors' knowledge there are no condition monitoring systems installed on structures in the US other than those that are part of research efforts. When installed, these condition-monitoring systems usually will require two types of measurements. For a class of structures such as office-buildings the monitoring system will be designed to compare measured response before, during, and after an extreme loading event such as an earthquake. The technology currently exists to make these types of measurements. For other structures, such as steel bridges where fatigue failures are possible, continuous monitoring is desirable. Continuous monitoring systems for civil engineering structures are currently in a research stage and are very expensive. Recently, Agbabian and Associates (Nigbor and Diehl, 1997), have started to market commercial data acquisition and analysis systems aimed at continuous *in situ* monitoring of structural condition. Annual maintenance costs for these systems are approximately ten percent of the initial system costs.

State-of-the-art in parameter estimation is presented by Hart and Yao, J. T. P. (1977), Liu, S. -C. and Yao, J. T. P. (1978), Mottorshead and Friswell (1993), Ghanem and Shinozuka (1995), Shinozuka and Ghanem (1995), and Doebling et al. (1996) present an overview of that development. It is the intent of this paper to summarize briefly structural parameter estimation with emphasis on application to civil engineering structures. This summary will attempt to highlight some of the technical challenges that have prevented this technology from becoming a more common practice in the civil engineering community.

CHALLENGE #1: DESIGN AND IMPLEMENTATION OF THE EXPERIMENT

The following issues are the primary challenges related to design and implementation of experiments in the field of parameter estimation for structural condition assessment:

1. Selection of excitation sources
2. Selection of type, number, and location of sensors
3. Data acquisition system calibration and integrity
4. Economy of testing.

Selection of excitation sources: The most basic question to consider when planning a test for the purposes of parameter estimation is the selection of an excitation source for the experiment, whether static or dynamic. Static testing is often more convenient, but dynamic testing can generally be done with lower force levels, which may be better for very large structures. The size of most civil engineering infrastructure makes it difficult to put sufficient amounts of energy into the structure in a controlled manner for the purpose of parameter estimation. Large eccentric mass shakers, hydraulic shakers, rocket motors, tensioned cables with explosive bolt cutters and explosives detonated in the ground have been used on various research projects. These excitation systems are very limited in terms of the input waveform that can be applied, are time consuming to setup, in some cases have significant safety concerns, and are very expensive. For continuous condition monitoring of civil infrastructure, measurement of structural response due to operating (ambient) excitation seems the only practical means. Dynamic system parameter estimation based on ambient vibration response has received considerably less study than parameter estimation based on controlled (measured) inputs.

Selection of type, number, and location of sensors: Sensor types are selected based on type of excitation such as static, dynamic, and temperature loading. Often, relative measurements are much easier to obtain than absolute measurements for full-scale structures. Examples of absolute measurements are acceleration, rotation, and strain. Examples of relative measurements are displacement and velocity. Another important question to answer while designing the test is the number and location of the measurement sensors. This is a combinatorial optimization problem and for large structures, there are millions of combinations of sensor locations and excitations. It is also important to select the measurement location, type, and number of sensors so that the measured response data is sensitive to the type and location of structures' condition changes that are of interest. In other words, the changes in the structural parameters must be observable by the sensors. Skelton and Delorenzo (1983), Haftka and Adelman (1985), and Kammer (1991) have developed methods for selection of noisy actuators and sensors locations on aerospace type of structures for modal identification or shape control. Sanayei et al. (1992, 1996) developed a heuristic method to select a noise-tolerant subset of measurements for error reduction in parameter estimates.

Data acquisition system calibration and integrity: When tests are conducted for the purpose of parameter estimation, many practical issues of data acquisition must be addressed. Measurement system calibration (not just sensor calibration), measurement system noise, identification of defective sensors, long term stability of the measurement system, and selection of sampling rates (including issues of aliasing, leakage, filtering, etc.) are some examples of these issues. Some of these issues are addressed in McConnell (1995), as well as commercial trade literature. Some of these topics become more significant when applications of continuous data acquisition over long periods of time are being considered. Most of these concerns are not the primary focus of current research projects. In addition to the traditional data acquisition issues there are issues of data compression and storage that must be addressed. These issues are particularly acute for continuous condition monitoring of civil infrastructure. However, as computer data storage becomes less expensive, these limitations are becoming less of an issue. Allemang (1993) discusses the cost of dynamic data acquisition, including both commercial hardware and software.

Economy of testing: With recent advances in computer hardware and software technology, data acquisition systems are more affordable and user friendly. However, the price of sensors (e.g., accelerometers and tilt-meters) due to low demand is still high. A recent development in data acquisition is use of wireless systems for condition assessment (Maser et al., 1997; Kiremidjian et al., 1997). An advantage is that there are no costs associated with wiring and related maintenance issues. Currently the cost of wireless testing is prohibitive, thus the current practice of structural parameter identification is primarily limited to one-of-a-kind research efforts performed by a limited number of institutions.

CHALLENGE #2: ERRORS IN MATHEMATICAL MODELING FOR PARAMETER ESTIMATION

Modeling error is one of the least understood areas of parameter estimation. This is not a major obstacle in structural analysis used for design of structural components. In order to remedy the modeling error in design, mathematical models are made to give upper bounds to components' forces and moments. In addition, a factor of safety is used to increase the reliability of the system. However, parameter estimation is an effort to reconcile the difference between a mathematical model and an associated physical structure. A priori mistakes in mathematical models that are not updated in parameter estimation pose a major source of uncertainty. This is often a smaller problem in a laboratory setup since the physical model, boundary conditions, connections, test setup, and data acquisition are better controlled. On the contrary, if the parameter estimation method is not based on a mathematical model, modeling error is not of any essence. The following issues are the primary challenges related to mathematical modeling errors that exist in the field of parameter estimation for structural condition assessment:

1. Existence of nonstructural members
2. Incomplete information about boundary conditions
3. Nonlinear structural response
4. Modeling of energy losses (damping)
5. Environmental variability
6. Economics of modeling for parameter estimation

Existence of nonstructural members: Nonstructural members increase the degree of uncertainty in the structural model. They can potentially alter the expected structural response. Therefore, parameter estimation using NDT data is easier to apply to skeleton type structures such as transmission towers, bridges, and parking garages rather than office or residential type buildings. Nonstructural components such as dry walls and partitions do not contribute much to the ultimate strength of the system. However, they increase the overall stiffness of the structure during the small-scale NDTs (Smith and Vance, 1996). Stiffer nonstructural members such as concrete masonry units (CMU) or rigid architectural walls can drastically alter the mode of failure in columns from bending to shear during earthquakes, causing a short column effect (Eagling, 1983). Parameter estimation may not provide satisfactory results if these failure modes are not modeled appropriately in the analytical FEM.

Incomplete information about boundary conditions: Modeling error can also stem from lack of understanding of structure’s boundary conditions. Structural connections are usually modeled only as hinged or continuos. However, most connections behave as partially restrained based on provided design details. Two elements, a partially restrained frame (PRF) and a soil-substructure super-element, have been developed by Sanayei et al. (1998) to improve connection behavior and include soil-structure interaction in parameter estimation. Such finite elements do not eliminate the modeling error, however, they can drastically improve the behavior of the mathematical model.

Nonlinear structural response: In full-scale testing often the physical structure has some degree of nonlinear behavior. Material nonlinearity (unlike geometric nonlinearity) is a major source of uncertainty in analytical models of full-scale structures if linear models are used. Frequently, structural models used for parameter estimation are linear to limit the computational effort. A primary source of parameter estimation research using nonlinearities uses earthquake engineering response data (Iwan and Cifuentes, 1986; Jayakumar and Beck, 1987). Major sources of nonlinearity are soil and concrete materials for civil structures. Soil stiffness may be considered linear only at very small strain levels. These strain levels are generally exceeded leading to a nonlinear response. Since different soils have different nonlinear behavior and the soil types at each site are not fully known, development of highly accurate models are not practical. In dynamic NDT, soil stiffness values are a function of the excitation frequency (Beredugo and Novak, 1972). Concrete by nature is also a nonlinear material. Both soil and concrete show hysteretic behavior in cyclic loading such as earthquakes and industrial machinery. Hystretic behavior leads to stiffness reduction and energy dissipation.

Modeling of energy losses: Modeling of damping is driven by mathematical convenience rather than in depth understanding of the physical phenomenon. Viscous damping does not necessarily capture the essence of energy dissipation. Structural or hysteretic damping is a better representation of energy dissipation. However, the response becomes frequency dependent. This is another source of uncertainty when linear mathematical models are adjusted to fit nonlinear dynamic field test data. Various energy loss mechanisms and models are reviewed by Ewins (1985). Hart and Vasudevan (1975) developed a damping estimation procedure and presented typical values for both elastic and inelastic building response analysis.

Environmental variability: Environmental variability is another challenge in parameter estimation of structures. Factors such as temperature, humidity, water table, and freeze and thaw conditions alter the long-term response of the structures. Wipf (1991) have correlated the long-term bridge movements to ambient temperature. Alampalli (1998) observed the vibration response of a simply-supported single-span bridge changes significantly, at below-freezing temperatures, to much like that of a bridge with partially fixed supports. Importance of these environmental factors can vary based on the type and location of the structure. If some of these factors are known to make a difference, they need to be included in the mathematical model. However, the above natural phenomena are not the simplest ones to model.

Economics of modeling for parameter estimation: Economics of modeling controls the level of attention to details. It is important to spend the time on portions of the model that changes in them create a larger impact in the response of the structure. If the mathematical model is finite element-based, one should decide how fine or coarse the model should be in each area. By making the model finer, if the element properties are unknown in that region, the size of the parameter estimation problem is increased. If the parameter estimation is optimization-based, much more computer time is required for parameter estimation.

CHALLENGE #3: PARAMETER ESTIMATION ERRORS

The following issues are the primary challenges to minimize the propagation of measurement noise into the parameter estimates:

1. Objective function, constraints, and optimization techniques
2. Degree of freedom mismatch
3. Selection of parameters to estimate and model order
4. Effects of relative uncertainty in measured data and model parameters

Objective function, constraints, and optimization techniques: The objective function, the constraints to be used in the optimization and the optimization technique itself are the most basic characteristics of a parameter estimation scheme. A physical model-based parameter-estimation scheme seeks to minimize an objective function that is some measure of error between the measured data and the anlytical model. An optimization algorithm is required to minimize the objective function under an appropriate set of constraints. In the case of a finite element model updating, the objective function is typically a nonlinear function of the model parameters, and thus a robust root-finding scheme such as Newton-Rhapson (Press et al., 1992) iteration is used to find a minimum of the objective function. Objective functions behave differently in presense of measurement noise due to degree of nonlinarity and characteristics of the mathematical formulation. The error sensitivity of any objective function used can be examined by simulation of the NDT using noisy measurements (Sanayei et al., 1992; 1996). The selection of constraint equations is also important in parameter estimation (Smith, 1992). Typically, these constraints are used to ensure that the proper structural connectivity is preserved, that the structural model matrices are symmetric and/or sparse, and that the model matrices remain positive definite or positive semidefinite after the parameter estimation scheme is applied. In some parameter estimation algorithms, the constraints are inherent in the formulation of the objective function and are not explicitly invoked during the optimization process.

Degree of freedom mismatch: Another aspect of parameter estimation that causes difficulties for implementation is the mismatch in degrees of freedom (DOF) between the analytical model and the experimentally measured data (i.e., incomplete set of measurements). Typically, a finite element model can contain tens or hundreds of thousands of DOF. However, test data sets usually contain tens, or at most hundreds, of measurement channels. Historically, the most common approach has been to implement Guyan (static) reduction (Guyan, 1965) to reduce the FEM size to the size of the test data set. However, this approach eliminates all but the lowest-frequency dynamic responses of the model, greatly reducing the achievable minimum value of the objective function and thus the overall accuracy of the estimated model parameters. An approach that has been used more widely in the research community is mode shape expansion. This procedure involves interpolating the values of the measured mode shapes at the model DOF that are not measured. Several procedures have been suggested for accomplishing this, such as those shown by (Zimmerman and Smith, 1992; Levine-West et al., 1994). Sanayei et al. (1998) developed a method that keeps all DOF and partition the analytical response to measured and unmeasured. Incomplete measured mode shapes were used to form a “modal equation error function” for stiffness and mass parameter estimation at the componet level. Another approach that has been proposed and proven effective is treating the unknown components in the mode shape vectors as additional unknowns in the model as shown by Farhat and Hemez (1993). One other approach that is closing the gap between the number of measured DOF and the number of FEM DOF is the use of measurement techniques such as laser doppler velocimeter systems that have the ability to make very spatially dense measurements. Robinson, et al. (1996) demonstrated the ability to make parameter estimates from a structure at over 2000 measurement locations using such a system.

Selection of parameters to estimate and model order: The selection of which model parameters should be estimated and the selection of the overall model order are both challenges currently slowing the implementation of parameter estimation technology. Selecting too many parameters to estimate may cause the problem to be underdetermined. On the other hand, selecting too few parameters to estimate may bias the results because a particular parameter that is very sensitive to the information in the data may be left out of the procedure. In the estimation of modal parameters, the selection of the model order for the parameter estimate can widely influence the results. Fitting too many modes may cause some “noise modes” (i.e. modes which are only fitting “noise,” because they are of extraneous model order) to appear in the result set. On the other hand, fitting too few modes may produce poor overall results or may fail to capture critical dynamic behavior. Low-order modal parameter estimation techniques, such as the Eigensystem Realization Algorithm (Juang and Pappa, 1985), bypass this problem by overspecifying the required order of the modal model, then filtering out those modes that fail to meet certain quality criteria. High-order modal parameter estimation techniques, such as the Rational Polynomial curve fit rely on prior knowledge of the number of modes in a certain frequency band (Richardson and Formenti, 1982).

Effects of relative uncertainty in measured data and model parameters: Another important issue in the parameter estimation process is the existence of uncertainty in the model form, the model initial parameters, and the experimental data. These uncertainties reduce the level of certainty in the parameter estimates. Brown (1987) developed an identification procedure to estimate model parameters using measured response as well as expert judgment. Fuzzy updating is used to supplement the problem with subjective information when numerical measurements are not available. Beck (1989) proposed a statistical system identification procedure that uses averaged response measurements to estimate the system parameters. By averaging the response of the system, the error due to measurement noise is reduced. Gangadharan et al. (1991) proposed a probabilistic system identification method to infer model parameters of flexible joints. The proposed method uses static responses of the structure to estimate stiffness coefficients as well as confidence bounds. Alvin (1997) presents a Bayesian method for finite element parameter estimation that uses covariance values on both the data and the model parameters, and produces covariance values on the resulting parameter estimates.

SUMMARY

Current primary challenges in parameter estimation for condition assessment of structures are reviewed for large civil-engineered structures. The first challenge in parameter estimation is design and implementation of the experiment. This includes selection of excitation sources; selection of type, number, and location of sensors; data acquisition system calibration and integrity; and economy of testing. The second challenge is reduction of errors in mathematical modeling. Major areas are existence of nonstructural members; incomplete information about boundary conditions; nonlinear structural response; modeling of energy losses (damping); environmental variability, and economics of modeling for parameter estimation. The third challenge is control of parameter estimation errors which includes objective function, constraints, and optimization techniques; degree of freedom mismatch; selection of parameters to estimate and model order; and effects of relative uncertainty in measured data and model parameters. This paper presents some of the above issues, both technical and economical, that must be addressed in order to make parameter estimation for structural condition assessment a practical reality.

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